818. Free flexural vibrations of a piezoelectric bimorph plate with periodic edge conditions

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Abstract. This work analyzes the vibrations of a fully-electroded annular piezoelectric bimorph plate with a free inner edge and an outer edge that is built-in with a periodicity. To this end, a variational formulation with the extensive use of Lagrange multipliers for a bimorph plate with polar orthorhombic symmetry is performed first. The mechanical displacement and the electric potential that must satisfy constraint conditions at the electrodes are expanded as the sums of powers in the thickness coordinate. The resulting piezoelectric bimorph plate equations are used along with the introduction of appropriate Lagrange multipliers to analyze the polar orthorhombic annular sectorial plates with free radial and inner circumferential edges, and an entirely built-in or free outer edge. The results are then combined to obtain the solutions for the mixed boundary value problem. The extended Hamilton's principle with the method of Lagrange multipliers is employed, followed by a Frobenius-type series expansion for solution functions. The eigensolutions are calculated from the resulting transcendental equation and compared with those obtained from an FEA to ensure the validity of the procedure.

Keywords: polar orthorhombic bimorph, annular plate, mixed boundary condition with periodicity, variational approximation procedure, Lagrange multipliers method.

Introduction

The vibration problem of annular and circular plates has been researched extensively using various theoretical or numerical approaches due to their wide range of applications in sensors and actuators. The free vibrations of annular and circular plates with various boundary conditions were well summarized by Leissa [1]. Free vibration problems of isotropic annular sector plates with simply supported radial edges were solved earlier using an exact method applied to a thin plate model [2]. Kobayashi et al. [3] obtained an analytical solution for the vibration of a Mindlin annular sector plate [4] with two simply supported radial edges and two free circumferential edges. Additionally, several numerical and semi- analytical studies have been performed to obtain approximate solutions for annular plates with various boundary conditions [5-16] by using the Rayleigh-Ritz method [5, 10], the Frobenius method [6], the finite difference method [7], the strip distribution transfer function method [11], the mode subtraction method [12], finite element method [13, 15], and so on. However, to the best of the authors' knowledge, variational treatment of the vibration problem of annular piezoelectric plates with mixed boundary conditions has not been reported.

This study analyzes the flexural vibrations of an annular polar orthorhombic bimorph plate with a mixed edge condition by means of variational approximation treatment [17-19]. The annular plate with a mixed outer circumferential edge with periodicity can be pictured as a combination of two types of sectorial plates with free inner edges, and one with a built-in outer edge and the other with a free outer edge. This problem applies to a bounded region containing several separated volumes with internal surfaces of discontinuity. Therefore, we must determine the dispersion relationships for both cases separately. These analyses are performed by employing the extended Hamilton principle with the Lagrange multipliers method within the framework of 2-D plate theory. The method of separation of variables is used, and the solution functions satisfying the differential equation in the radial direction are obtained by introducing a Frobenius-type series expansion [20, 21] about a regular point. The dispersion relationships are