828. Piecewise exact solution of nonlinear momentum conservation equation with unconditional stability for time increment

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Abstract. Exact solution is adopted for computation of the inviscid Burgers equation on finite difference grid. Initial condition and following computed values of the independent variable are assumed to be piecewisely linear between fixed grid points, and local exact solution is used to find the value at the next time step at each grid point. Comparisons of Piecewise Exact Solution Method (PESM), existing upwind scheme, and the analytic solution show that the present method is more accurate than the upwind scheme. The unconditional stability is a strong merit of this method and is shown with a test result.

Keywords: inviscid Burgers equation, piecewise exact solution method, upwind scheme, unconditional stability.

Introduction

Not only the advection equation but also the advection-diffusion equation has been intensively used to explain the flow of fluids in mathematical, physical, scientific, and engineering problems [1-3]. The advection equation becomes the advection-diffusion equation by adding diffusion terms with diffusion coefficients. Reversely, the advection equation can be thought as a partial equation containing advection phenomena only by cutting off the diffusion, or separating the partial differential operator into two, and getting two equations, i.e. the advection equation and the diffusion equation (fractional step method [4, 5] or operator-splitting method [6]). The advection equation often describes transport of a scalar property, e.g. concentration, temperature etc. A very specific case of the advection equation is the Burgers equation, when the dependent variable is the velocity itself [7, 8] instead of an arbitrary scalar property. The advection equation has been a focus of numerical modeling due to possible numerical diffusion or wiggles. Calculation of the inviscid Burgers equation is concerned here. Even though the Burgers equation is nonlinear, and different from linear advection equation, people have used similar numerical methods to solve the Burgers equation.

Fundamentally, the advection-diffusion equation:

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2} \tag{1}$$

is reduced to the advection equation:

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0 \tag{2}$$

The inviscid Burgers equation is a specific form of the advection equation, when $a = u$.