848. Nonlinear free vibration analysis of the functionally graded beams

H. Askari[1], D. Younesian[2], Z. Saadatnia[3], E. Esmailzadeh[4]
[1, 2, 3]Center of Excellence in Railway Transportation, School of Railway Engineering
Iran University of Science and Technology, Narmak, Tehran, 16846, Iran
[4]Faculty of Engineering and Applied Science, University of Ontario
Institute of Technology, Oshawa, Ontario, Canada
E-mail: [1]*askari.iust@gmail.com*, [2]*younesian@iust.ac.ir*, [3]*zia_saadat@rail.iust.ac.ir*, [4]*Ezadeh@uoit.ca*

(Received 10 April 2012; accepted 4 September 2012)

Abstract. Nonlinear natural oscillations of beams made from functionally graded material (FGM) are studied in this paper. The equation of motion is derived according to the Euler-Bernoulli beam theory and von Karman geometric nonlinearity. Subsequently, Galerkin's solution technique is applied to obtain the corresponding ordinary differential equation (ODE) for the FGM beam. This equation represents a kind of a nonlinear ODE containing quadratic and cubic nonlinear terms. This nonlinear equation is then solved by means of three efficient approaches. Homotopy perturbation method is applied at the first stage and the corresponding frequency-amplitude relationship is obtained. Frequency-amplitude formulation and Harmonic balance method are then employed and the consequent frequency responses are determined. In addition, Parameter Expansion Method is utilized for evaluating the nonlinear vibration of the system. A parametric study is then conducted to evaluate the influence of the geometrical and mechanical properties of the FGM beam on its frequency responses. Different types of material properties and boundary conditions are taken into account and frequency responses of the system are evaluated for different gradient indexes. The frequency ratio (nonlinear to linear natural frequency) is obtained in terms of the initial amplitude and compared for different materials and end conditions.

Keywords: frequency-amplitude relationship, FGM beams, nonlinear vibration, Euler-Bernoulli beam.

Introduction

The idea of functionally gradient materials was initiated by a team of researchers in Japan to avoid effect of thermal stresses. Thermal stresses are created as a result of direct bonding of metals and ceramics in high temperature applications. They can generate interface cracks, debonding at hetero-interfaces, and result in delimitation of the over-layer of the ceramics. Initially and originally, concept of FGM materials was employed for creating fuselage exterior and engine materials. Then, it was utilized for improving the figure of merit of thermoelectric materials [1]. FGMs were also employed in optoelectronics systems such as antireflective layers, fibers, GRIN lenses, and other passive elements made from dielectrics [2]. This concept was then widely employed to improve various mechanical and electrical systems. For instance, graded thermoelectric and dielectric elements [3], graded composite electrodes for solid oxide fuel cells [4], and piezoelectrically graded materials were employed for broadband ultrasonic transducers [5]. This idea was also applied for high current connectors [6]. Recently, vibration and dynamic analysis of functionally graded materials has attracted several researchers. Vibration of FGM cylindrical shells was analyzed by Loy et al. [7] in 1995. They have employed Love's shell theory and Rayleigh–Ritz method for obtaining strains displacements and eigenvalue governing equation. Dynamic response of initially stressed functionally graded rectangular thin plates was investigated by Yang and Shen [8]. The results of investigation are presented in Table 1. They conducted a parametric study on the effects of constituent volume fraction index, foundation stiffness, plate aspect ratio, the shape and duration of impulsive load as well as the initial membrane stresses on the dynamic response of the FGM plates. Large